Under Cover

Name: Key

You have just taken over as manager of a struggling umbrella company. Umbrellas are manufactured at a rate given by $G(t) = 20t - t^2$ umbrellas per hour for $0 \le t \le 14$, and t represents hours after the factory opens in the morning (6 AM). $G(t)$ is graphed below.

umbrellas/hour

hours

1. After a leisurely breakfast, you arrive at work at 9 AM.
   a. Write an expression that gives the number of umbrellas that have been produced before you even arrived.

      $\int_0^3 G(t)dt$

   b. Roughly how many umbrellas were produced during this time?

      $\frac{G(3)+G(0)}{2} \cdot 3 = \frac{51}{2} \cdot 3 = 76.5$

      roughly 77 umbrellas

2. At 12:30 PM you break for a long lunch.
   a. Write an expression that gives the number of umbrellas that have been produced that day up until your lunch break.

      $\int_0^{6.5} G(t)dt$

   b. Roughly how many umbrellas were produced during this time?

      $76.5 + \frac{G(3)+G(6.5)}{2} \cdot 3.5 = 319$ umbrellas

3. Write an equation involving an integral for a function $f(x)$, that gives the number of umbrellas produced x hours after the factory has opened.

   ind. variable

   $f(x) = \int_0^x G(\underline{t})\,d\underline{t}$

   f(x) is an accumulation function

4. Find $f'(8)$ and interpret your answer in the context of this problem.

   $f'(8) = G(8)$

   $\frac{d}{dx}(f(x)) = G(x)$

   $f'(8)$ is the rate at which umbrellas are produced at $t=8$ (2 PM) $G(8) = 20(8) - 8^2 = 96$ umbrellas per hour

5. When is $f(x)$ changing the fastest? How do you know?

   At $t=10$ (4 PM) because the rate of production is at a maximum

Topic 6.4—Accumulation Functions and the FTC

Important Ideas:

An accumulation function outputs the area under a curve from some starting value to x (the input).

Independent variable

$$F(x) = \int_{c}^{x} \underbrace{f(t)}_{\text{rate of accumulation}}\,dt$$

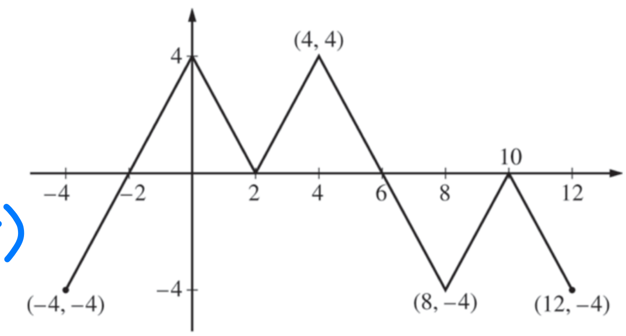
Starting value

FTC (Part 1): Rate of change of an accumulation function

$$\frac{d}{dx}\int_{c}^{x} f(t)\,dt = \frac{d}{dx}\left[F(x)\right] = f(x)$$

Check Your Understanding!

1. The graph of $f(t)$ is shown below. Let $h(x) = \int_{-2}^{x} f(t)dt$.



Graph of f

a. Find $h(2)$.

$$h(2) = \int_{-2}^{2} f(t)dt = \tfrac{1}{2}(4)(4) = 8$$

b. Find $h(8)$.

$$\int_{-2}^{8} f(t)dt = 8 + 8 - \tfrac{1}{2}(2)(4)$$
$$= 12$$

c. Find $h'(x)$.

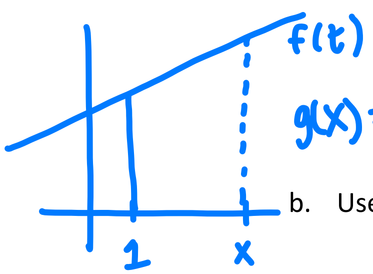
$$f(x)$$

2. Let $y = \int_{8}^{x} \sqrt{2 + e^{3t}}dt$. Find $\frac{dy}{dx}$.

$$\sqrt{2+e^{3x}}$$

3. Let $g(x) = \int_{1}^{x}(3t + 5)\,dt$

a. Find a formula for $g(x)$ that does not include integrals.



Trapezoid!

$$g(x) = \tfrac{1}{2}\left(f(1) + f(x)\right)(x-1) = \tfrac{1}{2}(8 + 3x + 5)(x-1)$$
$$= \tfrac{1}{2}(3x^2 + 10x - 13)$$

b. Use your answer in part a) to find $g'(x)$.

$$g'(x) = \tfrac{1}{2}(6x + 10) = 3x + 5$$

c. How do parts a) and b) illustrate the Fundamental Theorem of Calculus?

The derivative of an accumulation function is the original integrand function!

$$\frac{d}{dx}\int_{1}^{x}(3t+5)dt = 3x+5 \checkmark$$